 2013 NAME: KEY

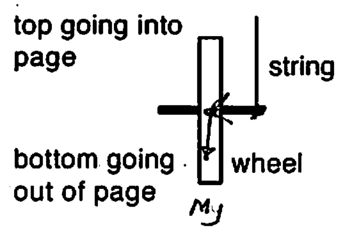
Test: *Rotational Mechanics*

Multiple Choice: *Choose the letter of the best answer. 3 points each.*

1. B What is the moment of inertia for a hoop of mass M and radius R spun about an axis perpendicular to the plane of the hoop and touching the edge of the hoop?

   a. $\frac{5}{2}MR^2$ b. $2MR^2$ c. $\frac{3}{2}MR^2$ d. MR^2 e. $\frac{1}{2}MR^2$

Questions 2 and 3 refer to the following:
A wheel is hanging from a string attached to one end of its axle. It is also rotating as shown in the diagram to the right.



2. D What is the direction of the angular velocity of the wheel?

   a. ↑ b. ↓ c. → d. ←
   e. out of paper. f. into the paper.

3. E What is the direction of the torque due to gravity on the wheel?

   a. ↑ b. ↓ c. → d. ← e. out of paper. f. into the paper.

4. A If a wheel is turning at 3.0 rad/s, the time it takes to complete one revolution is about:
   a. 2.1 s b. 1.3 s c. 1.0 s d. 0.67 s 4. 0.33 s

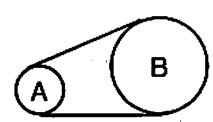
5. B A flywheel rotating at 12 rev/s is brought to rest in 6 s. The magnitude of the average angular acceleration in rad/s² of the wheel during this process is:
   a. 72 b. 4π c. 4 d. 2 e. 1/π

6. E A wheel starts from rest and has an angular acceleration that is given by $\alpha = 6t^2$. The angular velocity as a function of time for this motion is:
   a. $\omega = 6t^3$ b. $\omega = 3t^2$ c. $\omega = 12t$ d. $\omega = 6t$ e. $\omega = 2t^3$

7. E The rotational inertia of a disk about its axis is 0.70 kg·m². When a 2.0-kg weight is added to its rim, 0.40 m from the axis, the rotational inertia becomes:
   a. 0.38 kg·m² b. 0.54 kg·m² c. 0.70 kg·m² d. 0.86 kg·m² e. 1.02 kg·m²

8. E A top spinning on the floor precesses because the torque due to gravity, about the point of contact between the top and the floor is
   a. parallel to the angular momentum. b. parallel to the angular velocity.
   c. parallel to the axis of rotation. d. perpendicular to the floor.
   e. none of these.

9. D Wheel A and B are connected by a belt as shown at right. B has twice the radius of A. If the belt does not slip, what is true about the angular velocities of the wheels?



   a. A will rotate 4 times as fast as B.
   b. B will rotate 4 times as fast as A.
   c. B will rotate twice as fast as A.
   d. A will rotate twice as fast as B.
   e. They will have the same rotation rate.

10. C

11. B

12. D

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10. __C__ **A child is spinning on a merry-go-round. The child is 2 meters from the center, and is rotating at a constant speed of 0.5 rad/s. What is the magnitude of the linear acceleration of the child?**

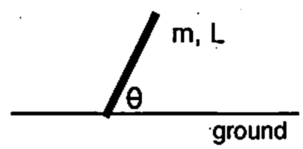
a. 0 m/s^2. b. 0.25 m/s^2. c. 0.5 m/s^2. d. 1.0 m/s^2. e. 2.0 m/s^2.

$r\omega^2 = (2)(.5)^2$

11. __B__ **A spinning figure skater will rotate faster if he pulls his arms in because**

a. His angular momentum and kinetic energy are conserved.
b. His angular momentum remains constant while his rotational inertia decreases.
c. His kinetic energy remains constant while his rotational inertia decreases.
d. His angular momentum increases while his rotational inertia decreases.

12. __D__ **What is the magnitude of the torque due to gravity on the leaning pole shown in the diagram below?**



a. $mLg\sin\theta$ b. $mLg\cos\theta$ c. $\frac{1}{2}mLg\sin\theta$ d. $\frac{1}{2}mLg\cos\theta$ e. none of those.

Problem Solving: *Show all work.* (12 @)

13. A wheel is initially spinning at 45 rpm. It then slows down at a constant rate, rotating through 50 radians in 20 seconds. How many more seconds will it take for the wheel to stop?

$\left(\frac{45 \text{ rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 4.71 \text{ rad/s} = \omega_i$

$\bar{\omega} = \frac{\theta}{t} = \frac{50}{20} = 2.5 \text{ rad/s}$

$\frac{4.71 + \omega}{2} = 2.5$

$4.71 + \omega = 5$

$\omega_f = 0.29$

$\alpha = \frac{.29 - 4.71}{20}$

$\alpha = -0.22 \text{ rad/s}^2$

$0 = (-.22)t + .29$

$t = \frac{.29}{.22}$

$t = 1.3 \text{ s}$

14. A solid sphere is at the top of a ramp of height h. How fast is the sphere going at the bottom of the ramp if it rolls without slipping?

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $I = \frac{2}{5}mr^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2$ $v = r\omega$

$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}v^2$

$v = \sqrt{\frac{10}{7}gh}$

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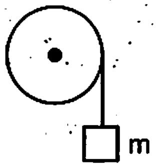
15. A 150 gram mass is hanging from a string around a pulley. The pulley has a radius of 25 cm. If the mass is accelerating down at 8 m/s², what is the moment of inertia of the pulley?



$mg - T = ma$

$rT = I\alpha$

$a = r\alpha$ $\quad 8 = .25\alpha$ $\quad \alpha = 32$

$T = mg - ma$ $\quad (1.5 - 1.2 = .3)$

$r(mg - ma) = I\alpha$

$rmg - rma = I\alpha$

$rmg - rmr\alpha = I\alpha$

$rmg - mr^2\alpha = I\alpha$

$(I + mr^2)\alpha = rmg$ (system!)

$\frac{rmg - rma}{\alpha} = I$

$I = \frac{rmg - rma}{a/r}$

$= \frac{mr^2(g-a)}{a}$

$(.25)(.3) = I\alpha$

$8 = .3\alpha$

$\alpha = 32$

$.08 = I\ 32$

$I = 2.34 \times 10^{-3}$

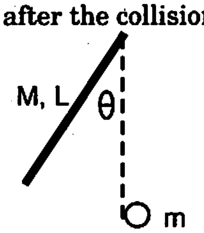
$I = .00234$

$\frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$ -10

$\tau = mg(r)$ -4

$T - mg = -ma$ -2

16. A 4 kg rod that is 1.2 meters long is free to swing about its end point. It is pulled back an angle of 40° and then released. At the exact bottom of its swing, it crashes and sticks to a (small) 2 kg mass that is at rest. The 2kg mass is What is the angular speed of the stick/mass immidieately after the collision?



$h = \frac{L}{2} - \frac{L}{2}\cos\theta$

$h = .6 - .6\cos 40 = .14\text{ m}$

$mgh = \frac{1}{2}I\omega^2$

$(4)(10)(.14) = \frac{1}{2}\left[\frac{1}{3}(4)(1.2)^2\right]\omega^2$ (1.92)

$5.61 = 0.96\ \omega^2$

$\omega^2 = 5.85$

$\omega = 2.42$

$I_1\omega_1 = I_2\omega_2$

$(1.92)(2.42) = (1.92 + 2(1.2)^2)\omega_2$ (2.88)

$4.64 = 4.80\ \omega_2$

$\boxed{\omega_2 = 0.97\text{ rad/s}}$

$\frac{1}{2}I\omega^2$ -4

mgh

$\Sigma\tau = I\alpha$ -4

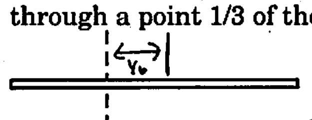
wrong h -2

$mv = (m+M)v$ -3

not conserved.

No L, -6

17. What is the moment of inertia for a thin rod of mass M and length L that is rotated about an axis through a point 1/3 of the way from its end?



(I) $\frac{1}{12}ML^2 + M\left(\frac{1}{6}L\right)^2$

$\frac{1}{12}ML^2 + \frac{1}{36}ML^2$

$\frac{3}{36} + \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$

$\boxed{= \frac{1}{9}ML^2}$

(II) $\frac{1}{3}\left[\frac{1}{3}M\right]\left[\frac{1}{3}L\right]^2 + \frac{1}{3}\left[\frac{2}{3}M\right]\left[\frac{2}{3}L\right]^2$

$= \frac{1}{81}ML^2 + \frac{8}{81}ML^2$

$\boxed{= \frac{1}{9}ML^2}$

(III) $\int_{-L/3}^{2L/3}\left(\frac{M}{L}\right)x^2\,dx$

$= \frac{M}{3L}x^3\Big|_{-L/3}^{2L/3}$

$= M\frac{1}{3L}\left[\left(\frac{2L}{3}\right)^3 - \left(-\frac{L}{3}\right)^3\right]$

$= M\frac{1}{3L}\left(\frac{8L^3}{27} + \frac{L^3}{27}\right)$

$= ML^2\frac{1}{3}\left(\frac{9}{27}\right) = \boxed{\frac{1}{9}ML^2}$

a